3.6 Mathematical Indices

index—an arbitrarily defined number that provides a measure of scale

In July 2001, CTV and *The Globe and Mail* ran a series of special reports on fitness in Canada. According to their surveys, 47.9% of Canadians are overweight with a body mass **index** (BMI) of 25 or more, while 15% are obese with a BMI in excess of 30. The body mass index is an example of a type of measure used for comparison. In this section, you will look at the BMI and other indices in order to see how they are used.

BODY MASS INDEX

Think about
Body Mass Index
What is the effect of dividing by the height squared as opposed to simply dividing by the height?

Health experts have adopted a mathematical formula called body mass index (BMI) to determine whether a person's mass puts them at risk for health problems. It is a crude measure of obesity that takes into account a person's height as well as mass. This helps to compensate for those who have a higher mass because they are tall. It is calculated by dividing a person's mass in kilograms by the square of their height in metres.

$$\text{BMI} = \frac{\text{mass}}{\text{height}^2}$$

Example 1 Calculating Body Mass Index

Calculate the BMI for the following individuals.

(a) height = 180 cm, mass = 70 kg **(b)** height = 165 cm, mass = 60 kg

Solution

(a) $\text{BMI} = \frac{70}{1.8^2} = 21.6 \text{ kg/m}^2$ **(b)** $\text{BMI} = \frac{60}{1.65^2} = 22.04 \text{ kg/m}^2$

Example 2 Using BMI to Calculate Mass

For a person whose height is 1.85 m, what mass would yield a BMI of 25? Of 30?

Solution

$\text{Mass} = 25 \times (1.85)^2$
$= 85.6 \text{ kg}$

$\text{Mass} = 30 \times (1.85)^2$
$= 102.7 \text{ kg}$

SLUGGING PERCENTAGE

Baseball is the most statistically analyzed sport in the world, with a number of indices used to measure the value of a player. In addition to a batting average, a player's offensive abilities can be measured by the slugging percentage. It takes into account not only the number of hits, but also the number of bases earned

(i.e., a double is worth two bases, a triple is worth three, and a home run is worth four). So, a player's slugging percentage may be calculated as

$$\text{SP} = \frac{\text{total bases}}{\text{at-bats}}$$

Example 3 Calculating Slugging Percentage

Here are some recent statistics for Barry Bonds. Compare his slugging percentage for these three years.

Year	AB	H	2B	3B	HR	AVG
1999	355	93	20	2	34	0.262
2000	480	147	28	4	49	0.306
2001	476	156	32	2	73	0.328

AB = at-bats; H = hits of any kind; 2B = two-base hits; 3B = three-base hits; HR = home runs (4 bases); AVG = $\frac{\text{H}}{\text{AB}}$

Solution

To determine the total number of bases (TB), use the formula TB = H + 2B + 2(3B) + 3(HR), since every double produces one extra base, every triple produces two extra bases, and every home run produces three extra bases.

In 1999, $\text{SP} = \frac{93 + (20) + 2(2) + 3(34)}{355}$

$= \frac{219}{355}$

$= 0.617$

In 2000, $\text{SP} = \frac{147 + (28) + 2(4) + 3(49)}{480}$

$= \frac{330}{480}$

$= 0.688$

In 2001, $\text{SP} = \frac{156 + (32) + 2(2) + 3(73)}{476}$

$= \frac{411}{476}$

$= 0.863$

CONSUMER PRICE INDEX

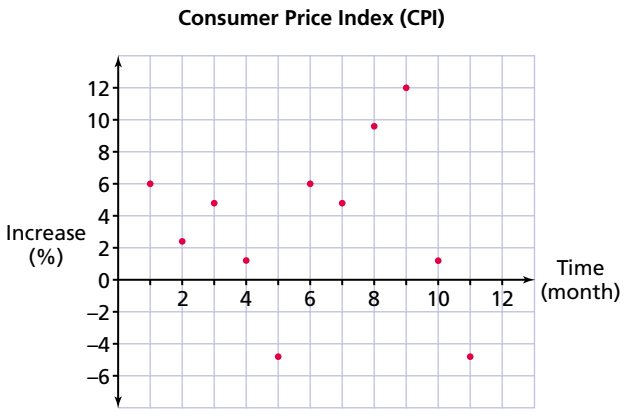
The consumer price index (CPI) provides a broad picture of the cost of living in Canada by comparing the costs of a wide variety of consumer goods, such as food, clothing, fuel, heating costs, transportation, shelter, and recreation. It is an important index since it is used to calculate increases in CPP payments to seniors, as well as increases to other plans indexed to the cost of living.

The consumer price index uses a formula that weights the influence of any particular item by how frequently it is purchased by the consumer. Food, which is purchased almost daily, is given a greater weight than clothing. Costs are compared to a base year, which at present is 1992. If the "shopping basket" of goods for 1992 had a value of $100, then, in the year 2000, it had a value of $113.50. This way, it is easy to express the increase as a percent.

This table shows the CPI for the 12-month period from August 2000 to July 2001.

Aug	Sept	Oct	Nov	Dec	Jan	Feb	Mar	Apr	May	June	July
113.9	114.4	114.6	115.0	115.1	114.7	115.2	115.6	116.4	117.4	117.5	117.1

A plot of these values shows a great deal of fluctuation.



Source: Statistics Canada

Technolink
To perform a moving average analysis in Fathom™, look in the numeric tools menu and click Analyze: Moving Average.

One way to smooth out fluctuations is to calculate a moving average.

moving average—an average of a number of consecutive points from time series data

MOVING AVERAGES

A moving average is used when time-series data show a great deal of fluctuation. It is used frequently in stock analysis where the price of a stock may vary greatly from day to day but the important feature is the long-term trend.

A moving average takes the average of the previous n values. For example, a three-day moving average reports the average stock price of the previous three days. Stock analysts may also use longer averages, such as 10-day, 30-day, 100-day, or 200-day averages.

Example 4 Calculating Moving Averages

Find the three-month moving average (annual percent interest) for the 12-month CPI increase given in the table at the top of this page.

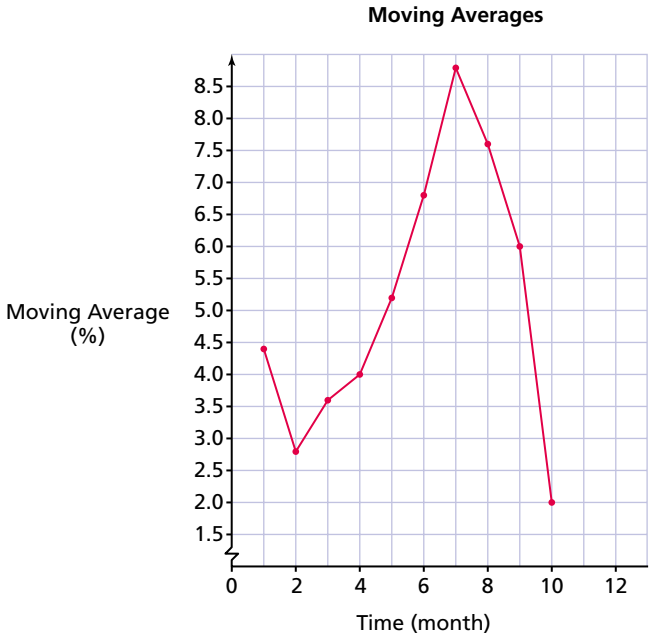
Solution

The first moving average that can be calculated is the average of the first three-month period (Aug, Sept, and Oct).

$$\frac{(114.4 - 113.9) \times 12 + (114.6 - 114.4) \times 12 + (115.0 - 114.6) \times 12}{3} = 4.4$$

The remaining results are shown in the table that follows. A plot of moving averages shows a general decrease over the first two months, followed by an increase, and then a decrease.

Months	Annual % Change
Aug, Sept, Oct	4.4%
Sept, Oct, Nov	2.8%
Oct, Nov, Dec	3.6%
Nov, Dec, Jan	4.0%
Dec, Jan, Feb	5.2%
Jan, Feb, Mar	6.8%
Feb, Mar, Apr	8.8%
Mar, Apr, May	7.6%
Apr, May, June	6.0%
May, June, July	2.0%



Think about
Annual % Increase

To calculate an annual percent increase, calculate the relative percent change for one month, and then multiply by 12. How does this give you the required percent?

Other moving averages will smooth out fluctuations over greater time periods. Many stock investment Web sites will provide 100- or 200-day moving averages.

KEY IDEAS

mathematical indices—an arbitrarily defined number that provides a measure of scale; determined by a mathematical calculation that combines various attributes of each case in order to make comparisons

body mass index (BMI)—an index designed to measure body mass

$$\text{BMI} = \frac{\text{mass}}{\text{height}^2}$$

slugging percentage—an index that measures the number of bases per at-bats

moving average—average of a number of consecutive points from time-series data; computed using the average of the previous *n* time periods (so you can calculate a three-day, a five-day, or an average of any number of days)

3.6 Exercises

A

1. Knowledge and Understanding

(a) Calculate the body mass index for the following individuals.
- **(i)** mass = 80 kg, height = 1.65 m
- **(ii)** mass = 70 kg, height = 1.70 m
- **(iii)** mass = 55 kg, height = 1.58 m
- **(iv)** mass = 90 kg, height = 2.10 m
- **(v)** mass = 100 kg, height = 2.15 m

(b) Rank the individuals in increasing order of their body mass index.

(c) How does the actual mass of each individual compare to the body mass index?

2. Calculate the mass that would yield a BMI of 20 or less for each of the following heights.

(a) 150 cm **(b)** 185 cm
(c) 170 cm **(d)** 200 cm

3. An athlete's BMI is 20. For each mass, how tall is the athlete?

(a) 65 kg **(b)** 80 kg
(c) 75 kg **(d)** 55 kg

4. Calculate the slugging percentage for these players on the 2001 Blue Jays.

Name	Games	2B	3B	AB	H	HR
Bush	78	11	1	271	83	3
Cruz Jr.	146	38	4	577	158	34
Delgado	162	31	1	574	160	39
Gonzalez	154	25	5	636	161	17
Stewart	155	44	7	640	202	12
Team Total		287	36	5663	1489	195

B

5. Investigate the most recent consumer price index.

(a) Calculate a moving average for the past 12 months.

(b) If there are any startling changes, can you associate them with any recent events that may have occurred?

Technolink
Up-to-date CPI data are available on the Statistics Canada Web site at **www.statcan.ca**.

6. Open the file **MaunaShort.ftm** on the textbook CD. Create a 12-month moving average to illustrate the general trend in CO_2 emissions.

7. **Application** The following table shows operating cost, seating capacity, and speed for several types of planes.

Plane	Seats	Speed (km/h)	Cost ($/h)
B747–400	396	538	6859
B747–100	447	520	6447
L–1011–100/200	310	495	3720
DC–10–10	289	500	5281
A300–600	249	473	5237
DC–10–30	265	520	6078
B767–300ER	214	495	3558
B757–200	186	465	2675
B767–200ER	181	488	3348

Source: Data have been extracted from Fathom Dynamic Statistics™, Key Curriculum Press.

(a) Create an index that calculates the cost to send 100 passengers on a 1000-km trip.

(b) Calculate this index for each type of airliner and rank them by increasing cost.

8. The following table shows prices in dollars of tickets and other items at some baseball parks.

Team	Adult	Child	Parking	Program	Cap	Soft Drink	Hot Dog
Atlanta	20.17	20.17	8.40	5.25	12.60	1.58	2.75
Baltimore	20.81	20.81	5.25	3.15	12.60	1.84	2.75
Boston	25.25	25.25	12.60	2.10	10.50	2.63	2.75
Ottawa	19.35	19.35	10.50	2.63	10.50	2.10	2.48
Colorado	16.58	16.58	8.40	5.25	12.60	2.89	3.03
Detroit	12.84	12.84	8.40	4.20	10.50	1.84	2.48
Florida	12.78	11.38	5.25	4.20	8.40	2.63	2.75
Montreal	9.85	8.68	6.94	3.48	10.42	1.73	2.00
New York	20.88	20.88	7.35	4.20	10.50	3.68	4.13
Toronto	27.45	17.33	8.33	3.48	7.98	1.44	1.55

(a) Create an index that calculates the cost of two adult tickets, two children's tickets, parking, two caps, four drinks, and four hot dogs.

(b) Calculate the value of this index for each park and rank the parks from least to most expensive.

(c) Your trip to the ballpark may not include all of the items listed in part (a). List the items you would purchase and calculate the index for your trip to each park.

9. Calculate a three-day moving average for the following stock price.

Day	Price ($)
1	9.99
2	10.99
3	11.72
4	11.82
5	11.56
6	15.38

Day	Price ($)
7	13.23
8	14.13
9	13.59
10	14.87
11	15.94

(a) Create a time-series graph showing the price and the moving average.
(b) If you are waiting until the price hits $20, how long might you wait?
(c) When was the actual price greater than the three-day average?
(d) Describe the effect of the price spike on day 6.

10. Design an index that you could use to compare soccer players on different teams. Could you use this index to compare male with female scores? Explain.

11. The following table lists the price of a stock over an interval of 14 business days.

Day	Price ($)
1	4.11
2	4.07
3	4.22
4	4.18
5	4.05
6	4.28
7	4.16

Day	Price ($)
8	4.22
9	4.29
10	4.25
11	4.25
12	4.51
13	4.37
14	4.21

(a) Use technology to create a three-day and a five-day moving average for these stock prices.
(b) Create a time-series graph and plot both sets of values.
(c) What effect does changing the number of days have on the display of the average?

12. The reading level of a book can be estimated by selecting 10 consecutive sentences and counting the number of words with more than three syllables. Find the square root of this total and add three.
(a) Estimate the reading level of this textbook.
(b) Compare this reading level with (i) a newspaper; (ii) a novel.
(c) What weaknesses do you perceive in this index?

13. **Communication** The United Nations calculates a Human Development Index (HDI) each year, largely for the purpose of measuring improving living conditions in the developing world. The HDI uses health (average life expectancy), knowledge (average education level), and economic well-being (average incomes) in measuring the quality of life in a country. Is this a fair evaluation? What other factors do you think are important?

14. Every year *Maclean's* magazine publishes its annual rankings of Canadian universities. It does this by creating an index that combines a number of factors, but the magazine does not reveal its actual formula for calculating the index. List factors that are important to you in considering a post-secondary school. Find *Maclean's* most recent ranking. Do you agree with their findings? Explain.

Chapter Problem

Comparing Marks

Many colleges in the United States use a numerical score called the Grade Point Average (GPA). It assigns a value between 0 and 4 for each final grade and computes an average weighted by the number of credits. Most of Justin's courses are full-credit courses, but he did take a half-credit course in Civics.

CP17. Use the table to the right to assign a numerical value for each of Justin's marks and calculate his weighted GPA.

Justin's Marks (all worth one credit unless noted)

English	86
Calculus	83 (2 credits)
Geometry	78
Data Management	88 (1.5 credits)
History	74
Physics	77
Civics	92 (0.5 credits)

Mark Range	Letter Grade	Numerical Value
93–100	A+	4.0
86–92	A	4.0
80–85	A–	3.7
77–79	B+	3.3
73–76	B	3.0
70–72	B–	2.7
67–69	C+	2.3
63–66	C	2.0
60–62	C–	1.7
57–59	D+	1.3
53–56	D	1.0
50–52	D–	0.7
<50	F	0